As these equations show, the component S_{11} of the linearized strain tensor can be used in exact finite-strain analyses in the special case of uniaxial strain. The components η_{11} and S_{11} are related by the exact direct and approximate inverse relations

$$\eta_{11} = S_{11} + S_{11}^2, \quad S_{11} = \eta_{11} - \eta_{11}^2 + 2\eta_{11}^3 + \dots . \tag{2.4}$$

The specific volume and density changes are related to the strain by the equations

$$S_{11} = v/v_{\mathrm{R}} - 1 = \rho_{\mathrm{R}}/\rho - 1, \tag{2.5}$$

where $\rho = 1/v$ is the mass density of the material at a given point and the subscript R designates evaluation in the reference state. When a body is subjected to uniaxial strain, its shape as well as its volume is changed and this change in shape is described by an angle γ (called the tensor component of the *shear strain*) that is related to the strain by the equation:

$$\gamma = (\tan^{-1}(S_{11} + 1)) - \tfrac{1}{4}\pi = \tfrac{1}{2}S_{11} + \dots . \tag{2.6}$$

Uniaxial strain is one of the simplest cases one can encounter and considerable effort is expended to ensure that it is the deformation produced in most shock-compression experiments.

Stress. The boundary loads and internal forces in continuous bodies can be represented by the symmetric tensor $\boldsymbol{t}$ called the *Cauchy stress tensor.* The component t_{ij} of this tensor is the component in the x_j direction of the force per unit area (in the deformed configuration) acting across a surface having its normal in the x_i direction. Because of its symmetry, $\boldsymbol{t}$ can be expressed in diagonal form. That is, the coordinates x can be chosen so that $t_{ij} = 0$ for all $i \neq j$. In this case the diagonal components are called *principal stresses.* Our sign convention has been chosen so that a positive value of one of the principal stresses corresponds to tension in the associated direction in the body.

When a slab of isotropic material, or a suitably oriented slab of lower symmetry, is loaded uniformly over one or both faces, a coordinate normal to the faces is an axis of principal stress. Suitably chosen directions in the plane of the slab are also directions of principal stress. When the response of the material is isotropic, the two lateral principal stresses are equal, so that

$$t_{11} \neq 0, \quad t_{22} = t_{33} \neq 0, \quad t_{ij} = 0, \quad i \neq j. \tag{2.7}$$

When these relations hold (with $t_{11} \neq t_{22}$), every plane in the body except those exactly parallel or perpendicular to the 1 axis is subjected to a shear stress. The magnitude of this stress is maximized on planes lying at 45° to the 1 axis, and its value on these planes is

$$\tau = \tfrac{1}{2}(t_{11} - t_{22}). \tag{2.8}$$

The pressure in the body is defined in terms of the average of the principal stresses:

$$p = -\tfrac{1}{3}(t_{11} + t_{22} + t_{33}) = -\tfrac{1}{3}(t_{11} + 2t_{22}). \tag{2.9}$$

From these formulae we see that the longitudinal stress component can be expressed in the form

$$t_{11} = -p + \tfrac{4}{3}\tau, \tag{2.10}$$

which shows how the applied load is borne in part by the pressure in the material and in part by its resistance to shear. The lateral stress components can be written $t_{22} = t_{33} = -p - \tfrac{2}{3}\tau$. In an experiment, the stress component t_{11} is subject to control by the imposed boundary conditions. The lateral stresses $t_{22} = t_{33}$ are not normally controlled and take whatever values are consistent with the state of uniaxial strain until they are altered by the arrival of waves from the lateral boundaries of the sample. It is this situation that leads to the statement that the sample is inertially confined.

Equations of balance. The forces in material bodies, and the motions they undergo, are constrained by the requirement that the equations representing the principles of conservation of mass and of balance of momentum and energy be satisfied. When the fields of interest are smooth enough, these relations can be expressed in terms of partial differential equations that take the forms

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0, \qquad \frac{\partial t_{11}}{\partial x} = \rho \dot{u}, \qquad \rho\left(\frac{\partial \varepsilon}{\partial t} + \frac{\partial \varepsilon}{\partial x} u\right) = t_{11}\frac{\partial u}{\partial x} \tag{2.11}$$

when restricted to motions of uniaxial strain in which body forces, thermal conduction and radiation are neglected. The quantity ε appearing in eq. $(2.11)_3$ is the *internal energy density*. Here, and subsequently, we have omitted the subscript 1 from vector components in the x_1 direction and from the coordinate itself.

The smoothness conditions required for the validity of equations (2.11) are satisfied for most values of x and t of interest in specific problems, but they may be violated on certain surfaces propagating through the material. One such surface, the *shock wave* (or simply *shock*), forms the principal subject of this review. A shock is a propagating surface across which the particle velocity and stress are discontinuous. At such a surface the differential equations of balance are replaced by the algebraic equations

$$[\![\rho(u - u_n)]\!] = 0, \qquad [\![\rho u(u - u_n) - t_{11}]\!] = 0, \qquad [\![\rho(\varepsilon + \tfrac{1}{2}u^2)(u - u_n) - t_{11}u]\!] = 0.$$

In these relations u_n is the velocity of normal displacement of the shock relative to the x_1 coordinate, and the brackets designate the jump in the enclosed quantity at the wave: $[\![\varphi]\!] = \varphi^- - \varphi^+$, where φ^+ and φ^- are limits of some function φ as the wave is approached from the front (the material into which it is advancing) and from the rear, respectively.

A less severe discontinuity than a shock is the acceleration wave, so called because the particle acceleration experiences a jump across the wave even though the particle velocity is continuous. Acceleration waves have been the subject of considerable theoretical [73C4, 76C1] and some experimental [74N4, 79G2] interest in recent years. Investigation of these waves is closely related to the present subject and offers considerable scientific promise.

The foregoing differential equations of motion and jump conditions have been expressed in terms of the independent variables x and t, the spatial or laboratory coordinates and time. In many cases it is convenient to deal with equivalent relations expressed in terms of the material coordinates. When this transformation is made, the field equations become

$$\partial t_{11}/\partial X = \rho_R \dot{u}, \qquad \rho_R \dot{\varepsilon} = t_{11}\partial u/\partial X, \tag{2.12}$$

with the density and displacement gradient standing in the relation (2.5). Similarly, the jump conditions for a shock wave take the form

$$[\![u + (\rho_R/\rho)U]\!] = 0, \qquad [\![t_{11} + \rho_R u U]\!] = 0, \qquad [\![\rho_R(\varepsilon + \tfrac{1}{2}u^2)U + t_{11}u]\!] = 0, \tag{2.13}$$

where U is the "Lagrangian" velocity of the wave, i.e., that relative to the X ($\equiv X_1$) coordinate. These latter relationships are often rewritten in the form

$$[\![\rho_R/\rho]\!] = -[\![u]\!]/U, \qquad [\![-t_{11}]\!] = \rho_R U[\![u]\!], \qquad [\![\varepsilon]\!] = \frac{1}{2\rho_R}(t_{11}^- + t_{11}^+)[\![\rho_R/\rho]\!], \tag{2.14}$$